

# Cognitive Radio: Brain-Empowered Wireless Communications

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*Invited Paper*

**Abstract**—Cognitive radio is viewed as a novel approach for improving the utilization of a precious natural resource: the radio electromagnetic spectrum.

The cognitive radio, built on a software-defined radio, is defined as an intelligent wireless communication system that is aware of its environment and uses the methodology of understanding-by-building to learn from the environment and adapt to statistical variations in the input stimuli, with two primary objectives in mind:

- highly reliable communication whenever and wherever needed;
- efficient utilization of the radio spectrum.

Following the discussion of interference temperature as a new metric for the quantification and management of interference, the paper addresses three fundamental cognitive tasks.

- 1) Radio-scene analysis.
- 2) Channel-state estimation and predictive modeling.
- 3) Transmit-power control and dynamic spectrum management.

This paper also discusses the emergent behavior of cognitive radio.

**Index Terms**—Awareness, channel-state estimation and predictive modeling, cognition, competition and cooperation, emergent behavior, interference temperature, machine learning, radio-scene analysis, rate feedback, spectrum analysis, spectrum holes, spectrum management, stochastic games, transmit-power control, water filling.

## I. INTRODUCTION

### A. Background

THE electromagnetic *radio spectrum* is a natural resource, the use of which by transmitters and receivers is licensed by governments. In November 2002, the Federal Communications Commission (FCC) published a report prepared by the Spectrum-Policy Task Force, aimed at improving the way in which this precious resource is managed in the United States [1]. The task force was made up of a team of high-level, multidisciplinary professional FCC staff—economists, engineers, and attorneys—from across the commission’s bureaus and offices. Among the task force major findings and recommendations, the second finding on page 3 of the report is rather revealing in the context of spectrum utilization:

“In many bands, spectrum access is a more significant problem than physical scarcity of spectrum, in large part due to legacy command-and-control regulation that limits the ability of potential spectrum users to obtain such access.”

Indeed, if we were to scan portions of the radio spectrum including the revenue-rich urban areas, we would find that [2]–[4]:

- 1) some frequency bands in the spectrum are largely unoccupied most of the time;
- 2) some other frequency bands are only partially occupied;
- 3) the remaining frequency bands are heavily used.

The underutilization of the electromagnetic spectrum leads us to think in terms of *spectrum holes*, for which we offer the following definition [2]:

*A spectrum hole is a band of frequencies assigned to a primary user, but, at a particular time and specific geographic location, the band is not being utilized by that user.*

Spectrum utilization can be improved significantly by making it possible for a secondary user (who is not being serviced) to access a spectrum hole unoccupied by the primary user at the right location and the time in question. *Cognitive radio* [5], [6], inclusive of software-defined radio, has been proposed as the means to promote the efficient use of the spectrum by exploiting the existence of spectrum holes.

But, first and foremost, what do we mean by cognitive radio? Before responding to this question, it is in order that we address the meaning of the related term “cognition.” According to the Encyclopedia of Computer Science [7], we have a three-point computational view of cognition.

- 1) *Mental states and processes* intervene between input stimuli and output responses.
- 2) The mental states and processes are described by *algorithms*.
- 3) The mental states and processes lend themselves to *scientific investigations*.

Moreover, we may infer from Pfeifer and Scheier [8] that the interdisciplinary study of cognition is concerned with exploring general principles of *intelligence* through a synthetic methodology termed *learning by understanding*. Putting these ideas together and bearing in mind that cognitive radio is aimed at improved utilization of the radio spectrum, we offer the following definition for cognitive radio.

*Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside*

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world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind:

- highly reliable communications whenever and wherever needed;
- efficient utilization of the radio spectrum.

Six key words stand out in this definition: awareness,<sup>1</sup> intelligence, learning, adaptivity, reliability, and efficiency. Implementation of this far-reaching combination of capabilities is indeed feasible today, thanks to the spectacular advances in digital signal processing, networking, machine learning, computer software, and computer hardware.

In addition to the cognitive capabilities just mentioned, a cognitive radio is also endowed with *reconfigurability*.<sup>2</sup> This latter capability is provided by a platform known as *software-defined radio*, upon which a cognitive radio is built. Software-defined radio (SDR) is a practical reality today, thanks to the convergence of two key technologies: digital radio, and computer software [11]–[13].

### B. Cognitive Tasks: An Overview

For reconfigurability, a cognitive radio looks naturally to software-defined radio to perform this task. For other tasks of a cognitive kind, the cognitive radio looks to signal-processing and machine-learning procedures for their implementation. The cognitive process starts with the passive *sensing of RF stimuli* and culminates with *action*.

In this paper, we focus on three *on-line* cognitive tasks<sup>3</sup>:

- 1) Radio-scene analysis, which encompasses the following:
  - estimation of interference temperature of the radio environment;
  - detection of spectrum holes.
- 2) Channel identification, which encompasses the following:
  - estimation of channel-state information (CSI);
  - prediction of channel capacity for use by the transmitter
- 3) Transmit-power control and dynamic spectrum management.

Tasks 1) and 2) are carried out in the receiver, and task 3) is carried out in the transmitter. Through interaction with the RF

<sup>1</sup>According to Fette [10], the awareness capability of cognitive radio embodies awareness with respect to the transmitted waveform, RF spectrum, communication network, geography, locally available services, user needs, language, situation, and security policy.

<sup>2</sup>Reconfigurability provides the basis for the following features [13].

- Adaptation of the radio interface so as to accommodate variations in the development of new interface standards.
- Incorporation of new applications and services as they emerge.
- Incorporation of updates in software technology.
- Exploitation of flexible heterogeneous services provided by radio networks.

<sup>3</sup>Cognition also includes language and communication [9]. The cognitive radio's language is a set of signs and symbols that permits different internal constituents of the radio to communicate with each other. The cognitive task of language understanding is discussed in Mitola's Ph.D. dissertation [6]; for some further notes, see Section XII-A.

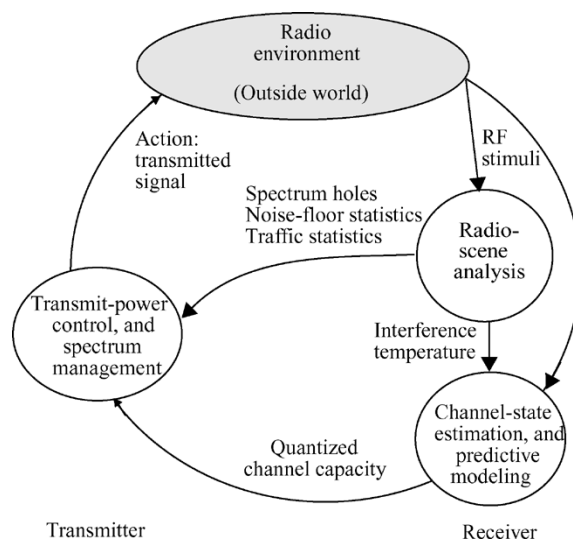


Fig. 1. Basic cognitive cycle. (The figure focuses on three fundamental cognitive tasks.)

environment, these three tasks form a cognitive cycle,<sup>4</sup> which is pictured in its most basic form in Fig. 1.

From this brief discussion, it is apparent that the cognitive module in the transmitter must work in a harmonious manner with the cognitive modules in the receiver. In order to maintain this harmony between the cognitive radio's transmitter and receiver at all times, we need a *feedback channel* connecting the receiver to the transmitter. Through the feedback channel, the receiver is enabled to convey information on the performance of the forward link to the transmitter. The cognitive radio is, therefore, by necessity, an example of a *feedback communication system*.

One other comment is in order. A broadly defined cognitive radio technology accommodates a *scale of differing degrees of cognition*. At one end of the scale, the user may simply pick a spectrum hole and build its cognitive cycle around that hole. At the other end of the scale, the user may employ multiple implementation technologies to build its cognitive cycle around a wideband spectrum hole or set of narrowband spectrum holes to provide the best expected performance in terms of spectrum management and transmit-power control, and do so in the most highly secure manner possible.

### C. Historical Notes

Unlike conventional radio, the history of which goes back to the pioneering work of Guglielmo Marconi in December 1901, the development of cognitive radio is still at a conceptual stage. Nevertheless, as we look to the future, we see that cognitive radio has the potential for making a significant difference to the way in which the radio spectrum can be accessed with improved utilization of the spectrum as a primary objective. Indeed, given

<sup>4</sup>The idea of a cognitive cycle for cognitive radio was first described by Mitola in [5]; the picture depicted in that reference is more detailed than that of Fig. 1. The cognitive cycle of Fig. 1 pertains to a one-way *communication path*, with the transmitter and receiver located in two different places. In a *two-way communication* scenario, we have a *transceiver* (i.e., combination of transmitter and receiver) at each end of the communication path; all the cognitive functions embodied in the cognitive cycle of Fig. 1 are built into each of the two transceivers.

its potential, cognitive radio can be justifiably described as a “disruptive, but unobtrusive technology.”

The term “cognitive radio” was coined by Joseph Mitola.<sup>5</sup> In an article published in 1999, Mitola described how a cognitive radio could enhance the flexibility of personal wireless services through a new language called the *radio knowledge representation language* (RKRL) [5]. The idea of RKRL was expanded further in Mitola’s own doctoral dissertation, which was presented at the Royal Institute of Technology, Sweden, in May 2000 [6]. This dissertation presents a conceptual overview of cognitive radio as an exciting multidisciplinary subject.

As noted earlier, the FCC published a report in 2002, which was aimed at the changes in technology and the profound impact that those changes would have on spectrum policy [1]. That report set the stage for a workshop on cognitive radio, which was held in Washington, DC, May 2003. The papers and reports that were presented at that Workshop are available at the Web site listed under [14]. This Workshop was followed by a Conference on Cognitive Radios, which was held in Las Vegas, NV, in March 2004 [15].

#### D. Purpose of this Paper

In a short section entitled “Research Issues” at the end of his Doctoral Dissertation, Mitola goes on to say the following [6]:

“How do cognitive radios learn best? merits attention.”

The exploration of learning in cognitive radio includes the internal tuning of parameters and the external structuring of the environment to enhance machine learning. Since many aspects of wireless networks are artificial, they may be adjusted to enhance machine learning. This dissertation did not attempt to answer these questions, but it frames them for future research.”

The primary purpose of this paper is to build on Mitola’s visionary dissertation by presenting detailed expositions of signal-processing and adaptive procedures that lie at the heart of cognitive radio.

#### E. Organization of this Paper

The remaining sections of the paper are organized as follows.

- Sections II–V address the task of radio-scene analysis, with Section II introducing the notion of interference temperature as a new metric for the quantification and management of interference in a radio environment. Section III reviews nonparametric spectrum analysis with emphasis on the multitaper method for spectral estimation, followed by Section IV on application of the multitaper method to noise-floor estimation. Section V discusses the related issue of spectrum-hole detection.
- Section VI discusses channel-state estimation and predictive modeling.
- Sections VII–X are devoted to multiuser cognitive radio networks, with Sections VII and VIII reviewing stochastic games and highlighting the processes of cooperation and competition that characterize multiuser networks. Section IX discusses an iterative water-filling (WF) procedure for distributed transmit-power control.

<sup>5</sup>It is noteworthy that the term “software-defined radio” was also coined by Mitola.

Section X discusses the issues that arise in dynamic spectrum management, which is performed hand-in-hand with transmit-power control.

- Section XI discusses the related issue of emergent behavior that could arise in a cognitive radio environment.
- Section XII concludes the paper and highlights the research issues that merit attention in the future development of cognitive radio.

## II. INTERFERENCE TEMPERATURE

Currently, the radio environment is *transmitter-centric*, in the sense that the transmitted power is designed to approach a prescribed noise floor at a certain distance from the transmitter. However, it is possible for the RF noise floor to rise due to the unpredictable appearance of new sources of interference, thereby causing a progressive degradation of the signal coverage. To guard against such a possibility, the FCC Spectrum Policy Task Force [1] has recommended a paradigm shift in interference assessment, that is, a shift away from largely fixed operations in the transmitter and toward *real-time interactions between the transmitter and receiver in an adaptive manner*. The recommendation is based on a new metric called the *interference temperature*,<sup>6</sup> which is intended to quantify and manage the sources of interference in a radio environment. Moreover, the specification of an *interference-temperature limit* provides a “worst case” characterization of the RF environment in a particular frequency band and at a particular geographic location, where the receiver could be expected to operate satisfactorily.

The recommendation is made with two key benefits in mind.<sup>7</sup>

- 1) The interference temperature at a receiving antenna provides an accurate measure for the acceptable level of RF interference in the frequency band of interest; any transmission in that band is considered to be “harmful” if it would increase the noise floor above the interference-temperature limit.
- 2) Given a particular frequency band in which the interference temperature is not exceeded, that band could be made available to unserved users; the interference-temperature limit would then serve as a “cap” placed on potential RF energy that could be introduced into that band.

For obvious reasons, regulatory agencies would be responsible for setting the interference-temperature limit, bearing in mind the condition of the RF environment that exists in the frequency band under consideration.

What about the unit for interference temperature? Following the well-known definition of equivalent noise temperature of a receiver [20], we may state that the interference temperature is measured in *degrees Kelvin*. Moreover, the interference-temperature limit  $T_{\max}$  multiplied by *Boltzmann’s constant*

<sup>6</sup>We may also introduce the concept of *interference temperature density*, which is defined as the interference temperature per capture area of the receiving antenna [16]. The interference temperature density could be made independent of the receiving antenna characteristics through the use of a reference antenna.

In a historical context, the notion of radio noise temperature is discussed in the literature in the context of microwave background, and also used in the study of solar radio bursts [17], [18].

<sup>7</sup>Inference temperature has aroused controversy. In [19], the National Association for Amateur Radio presents a critique of this metric.

$k = 1.3807 \times 10^{-23}$  joules per degree Kelvin yields the corresponding upper limit on permissible power spectral density in a frequency band of interest, and that density is measured in joules per second or, equivalently, watts per hertz.

### III. RADIO-SCENE ANALYSIS: SPACE-TIME PROCESSING CONSIDERATIONS

The stimuli generated by radio emitters are *nonstationary spatio-temporal signals* in that their statistics depend on both time and space. Correspondingly, the passive task of radio-scene analysis involves space-time processing, which encompasses the following operations.

- 1) Two adaptive, spectrally related functions, namely, estimation of the interference temperature, and detection of spectrum holes, both of which are performed at the receiving end of the system. (Information obtained on these two functions, sent to the transmitter via a feedback channel, is needed by the transmitter to carry out the joint function of active transmit-power control and dynamic spectrum management.)
- 2) Adaptive beamforming for interference control, which is performed at both the transmitting and receiving ends of the system in a complementary fashion.

#### A. Time-Frequency Distribution

Unfortunately, the statistical analysis of nonstationary signals, exemplified by RF stimuli, has had a rather mixed history. Although the general second-order theory of nonstationary signals was published during the 1940s by Loève [21], [22], it has not been applied nearly as extensively as the theory of stationary processes published only slightly previously and independently by Wiener and Kolmogorov.

To account for the nonstationary behavior of a signal, we have to include time (implicitly or explicitly) in a statistical description of the signal. Given the desirability of working in the frequency domain for well-established reasons, we may include the effect of time by adopting a *time-frequency distribution* of the signal. During the last 25 years, many papers have been published on various estimates of time-frequency distributions; see, for example, [23] and the references cited therein. In most of this work, however, the signal is assumed to be deterministic. In addition, many of the proposed estimators of time-frequency distributions are constrained to match time and frequency marginal density conditions. However, the frequency marginal distribution is, except for a scaling factor, just the periodogram of the signal. At least since the early work of Rayleigh [24], it has been known that *the periodogram is a badly biased and inconsistent estimator of the power spectrum*. We, therefore, do not consider matching marginal distributions to be important. Rather, we advocate a stochastic approach to time-frequency distributions which is rooted in the works of Loève [21], [22] and Thomson [25], [26].

For the stochastic approach, we may proceed in one of two ways.

- 1) The incoming RF stimuli are sectioned into a continuous sequence of successive bursts, with each burst being short enough to justify pseudostationarity and yet long enough to produce an accurate spectral estimate.
- 2) Time and frequency are considered jointly under the Loève transform.

Approach 1) is well suited for wireless communications. In any event, we need a nonparametric method for spectral estimation that is both accurate and principled. For reasons that will become apparent in what follows, multitaper spectral estimation is considered to be the method of choice.

#### B. Multitaper Spectral Estimation

In the spectral estimation literature, it is well known that the estimation problem is made difficult by the *bias-variance dilemma*, which encompasses the interplay between two points.

- Bias of the power-spectrum estimate of a time series, due to the sidelobe leakage phenomenon, is reduced by *tapering* (i.e., *windowing*) the time series.
- The cost incurred by this improvement is an increase in variance of the estimate, which is due to the loss of information resulting from a reduction in the effective sample size.

How can we resolve this dilemma by mitigating the loss of information due to tapering? The answer to this fundamental question lies in the principled use of *multiple orthonormal tapers* (*windows*),<sup>8</sup> an idea that was first applied to spectral estimation by Thomson [26]. The idea is embodied in the *multitaper spectral estimation procedure*.<sup>9</sup> Specifically, the procedure linearly expands the part of the time series in a fixed bandwidth  $f - W$  to  $f + W$  (centered on some frequency  $f$ ) in a special family of sequences known as the *Slepian sequences*.<sup>10</sup> The remarkable property of Slepian sequences is that their Fourier transforms have the *maximal energy concentration* in the bandwidth  $f - W$  to  $f + W$  under a finite sample-size constraint. This property, in turn, allows us to trade spectral resolution for improved spectral characteristics, namely, reduced variance of the spectral estimate without compromising the bias of the estimate.

Given a time series  $\{x_t\}_{t=1}^N$ , the multitaper spectral estimation procedure determines two things.

- 1) An orthonormal sequence of  $K$  Slepian tapers denoted by  $\{w_t^{(k)}\}_{t=1}^N$ .

<sup>8</sup>Another method for addressing the bias-variance dilemma involves dividing the time series into a set of possible overlapping segments, computing a periodogram for each tapered (windowed) segment, and then averaging the resulting set of power spectral estimates, which is what is done in *Welch's method* [27]. However, unlike the principled use of multiple orthogonal tapers, Welch's method is rather ad hoc in its formulation.

<sup>9</sup>In the original paper by Thomson [36], the multitaper spectral estimation procedure is referred to as the *method of multiple windows*. For detailed descriptions of this procedure, see [26], [28] and the book by Percival and Walden [29, Ch. 7]. The *Signal Processing Toolbox* [30] includes the MATLAB code for Thomson's multitaper method and other nonparametric, as well as parametric methods of spectral estimation.

<sup>10</sup>The Slepian sequences are also known as *discrete prolate spheroidal sequences*. For detailed treatment of these sequences, see the original paper by Slepian [31], the appendix to Thomson's paper [26], and the book by Percival and Walden [29, Ch. 8].

- 2) The associated eigenspectra defined by the Fourier transforms

$$Y_k(f) = \sum_{t=1}^N w_t^{(k)} x(t) e^{-j2\pi ft}, \quad k = 0, 1, \dots, K-1. \quad (1)$$

The energy distributions of the eigenspectra are concentrated inside a *resolution bandwidth*, denoted by  $2W$ . The *time-bandwidth product*

$$p = 2NW \quad (2)$$

defines the *degrees of freedom* available for controlling the variance of the spectral estimator. The choice of parameters  $K$  and  $p$  provides a tradeoff between spectral resolution and variance.<sup>11</sup> A natural spectral estimate, based on the first few eigenspectra that exhibit the least sidelobe leakage, is given by

$$\hat{S}(f) = \frac{\sum_{k=0}^{K-1} \lambda_k(f) |Y_k(f)|^2}{\sum_{k=0}^{K-1} \lambda_k(f)} \quad (3)$$

where  $\lambda_k$  is the eigenvalue associated with the  $k$ th eigenspectrum. Two points are noteworthy.

- 1) The denominator in (3) makes the estimate  $\hat{S}(f)$  unbiased.
- 2) Provided that we choose  $K = 2NW - 1$ , then the eigenvalue  $\lambda_k$  is close to unity, in which case

$$K \approx \sum_{k=0}^{K-1} \lambda_k.$$

Moreover, the spectral estimate  $\hat{S}(f)$  can be improved by the use of “adaptive weighting,” which is designed to minimize the presence of broadband leakage in the spectrum [26], [28].

It is important to note that in [33], Stoica and Sundin show that the multitaper spectral estimation procedure can be interpreted as an “approximation” of the *maximum-likelihood* power spectrum estimator. Moreover, they show that for wideband signals, the multitaper spectral estimation procedure is “nearly optimal” in the sense that it almost achieves the Cramér–Rao bound for a nonparametric spectral estimator. Most important, unlike the maximum-likelihood spectral estimator, the multitaper spectral estimator is computationally feasible.

### C. Adaptive Beamforming for Interference Control

Spectral estimation accounts for the temporal characteristic of RF stimuli. To account for the spatial characteristic of RF stimuli, we resort to the use of *adaptive beamforming*.<sup>12</sup> The motivation for so doing is *interference control* at the cognitive radio receiver, which is achieved in two stages.

<sup>11</sup>For an estimate of the variance of a multitaper spectral estimator, we may use a resampling technique called *Jackknifing* [32]. The technique bypasses the need for finding an exact analytic expression for the probability distribution of the spectral estimator, which is impractical because time-series data (e.g., stimuli produced by the radio environment) are typically nonstationary, non-Gaussian, and frequently contain outliers. Moreover, it may be argued that the multitaper spectral estimation procedure results in nearly uncorrelated coefficients, which provides further justification for the use of jackknifing.

<sup>12</sup>Adaptive beamformers, also referred to as adaptive antennas or smart antennas, are discussed in the books [34]–[37].

In the first stage of interference control, the transmitter exploits *geographic awareness* to focus its radiation pattern along the direction of the receiver. Two beneficial effects result from beamforming in the transmitter.

- 1) At the transmitter, power is preserved by avoiding radiation of the transmitted signal in all directions.
- 2) Assuming that every cognitive radio transmitter follows a strategy similar to that summarized under point 1), interference at the receiver due to the actions of other transmitters is minimized.

At the receiver, beamforming is performed for the *adaptive cancellation* of residual interference from known transmitters, as well as interference produced by other unknown transmitters. For this purpose, we may use a robustified version of the *generalized sidelobe canceller* [38], [39], which is designed to protect the target RF signal and place nulls along the directions of interferers.

## IV. INTERFERENCE-TEMPERATURE ESTIMATION

With cognitive radio being receiver-centric, it is necessary that the receiver be provided with a reliable spectral estimate of the interference temperature. We may satisfy this requirement by doing two things.

- 1) *Use the multitaper method to estimate the power spectrum of the interference temperature due to the cumulative distribution of both internal sources of noise and external sources of RF energy.* In light of the findings reported in [33], this estimate is near-optimal.
- 2) *Use a large number of sensors to properly “sniff” the RF environment, wherever it is feasible.* The large number of sensors is needed to account for the spatial variation of the RF stimuli from one location to another.

The issue of multiple-sensor permissibility is raised under point 2) because of the diverse ways in which wireless communications could be deployed. For example, in an indoor building environment and communication between one building and another, it is feasible to use multiple sensors (i.e., antennas) placed at strategic locations in order to improve the reliability of interference-temperature estimation. On the other hand, in the case of an ordinary mobile unit with limited real estate, the interference-temperature estimation may have to be confined to a single sensor. In what follows, we describe the multiple-sensor scenario, recognizing that it includes the single-sensor scenario as a special case.

Let  $M$  denote the total number of sensors deployed in the RF environment. Let  $Y_k^{(m)}(f)$  denote the  $k$ th eigenspectrum computed by the  $m$ th sensor. We may then construct the  $M$ -by- $K$  spatio-temporal complex-valued matrix

$$\mathbf{A}(f) = \begin{bmatrix} w_1 Y_1^{(1)}(f) & w_1 Y_2^{(1)}(f) & \dots & w_1 Y_K^{(1)}(f) \\ w_2 Y_1^{(2)}(f) & w_2 Y_2^{(2)}(f) & \dots & w_2 Y_K^{(2)}(f) \\ \vdots & \vdots & \ddots & \vdots \\ w_M Y_1^{(M)}(f) & w_M Y_2^{(M)}(f) & \dots & w_M Y_K^{(M)}(f) \end{bmatrix} \quad (4)$$

where each column is produced using stimuli sensed at a different gridpoint, each row is computed using a different Slepian

taper, and the  $\{w_m\}_{m=1}^M$  represent variable weights accounting for relative areas of gridpoints, as described in [40].

Each entry in the matrix  $\mathbf{A}(f)$  is produced by two contributions, one due to additive internal noise in the sensor and the other due to the incoming RF stimuli. Insofar as radio-scene analysis is concerned, however, the primary contribution of interest is that due to RF stimuli. An effective tool for denoising is the *singular value decomposition* (SVD), the application of which to the matrix  $\mathbf{A}(f)$  yields the decomposition [41]

$$\mathbf{A}(f) = \sum_{k=0}^{K-1} \sigma_k(f) \mathbf{u}_k(f) \mathbf{v}_k^\dagger(f) \quad (5)$$

where  $\sigma_k(f)$  is the  $k$ th *singular value* of matrix  $\mathbf{A}(f)$ ,  $\mathbf{u}_k(f)$  is the associated *left singular vector*, and  $\mathbf{v}_k(f)$  is the associated *right singular vector*; the superscript  $\dagger$  denotes Hermitian transposition. In analogy with principal components analysis, the decomposition of (5) may be viewed as one of *principal modulations* produced by the external RF stimuli. According to (5), the singular value  $\sigma_k(f)$  scales the  $k$ th principal modulation of matrix  $\mathbf{A}(f)$ .

Forming the  $K$ -by- $K$  matrix product  $\mathbf{A}^\dagger(f)\mathbf{A}(f)$ , we find that the entries on the main diagonal of this product, except for a scaling factor, represent the eigenspectrum due to each of the Slepian tapers, spatially averaged over the  $M$  sensors. Let the singular values of matrix  $\mathbf{A}(f)$  be ordered  $|\sigma_0(f)| \geq |\sigma_1(f)| \geq \dots \geq |\sigma_{K-1}(f)| < 0$ . The  $k$ th eigenvalue of  $\mathbf{A}^\dagger(f)\mathbf{A}(f)$  is  $|\sigma_k(f)|^2$ . We may then make the following statements.

- 1) The *largest eigenvalue*, namely,  $|\sigma_0(f)|^2$ , provides an estimate of the interference temperature, except for a constant. This estimate may be improved by using a linear combination of the largest two or three eigenvalues:  $|\sigma_k(f)|^2$ ,  $k = 0, 1, 2$ .
- 2) The left singular vectors, namely, the  $\mathbf{u}_k(f)$ , give the *spatial distribution of the interferers*.
- 3) The right singular vectors, namely, the  $\mathbf{v}_k(f)$ , give the *multitaper coefficients* for the interferers' waveform.

To summarize, *multitaper spectral estimation combined with singular value decomposition* provides an effective procedure for estimating the power spectrum of the noise floor in an RF environment. A cautionary note, however, is in order: the procedure is computationally intensive but nevertheless manageable. In particular, the computation of eigenspectra followed by singular value decomposition would have to be repeated at each frequency of interest.

## V. DETECTION OF SPECTRUM HOLES

In passively sensing the radio scene and thereby estimating the power spectra of incoming RF stimuli, we have a basis for classifying the spectra into three broadly defined types, as summarized here.

- 1) *Black spaces*, which are occupied by high-power "local" interferers some of the time.
- 2) *Grey spaces*, which are partially occupied by low-power interferers.

- 3) *White spaces*, which are free of RF interferers except for *ambient noise*, made up of natural and artificial forms of noise, namely:
  - broadband thermal noise produced by external physical phenomena such as solar radiation;
  - transient reflections from lightening, plasma (fluorescent) lights, and aircraft;
  - impulsive noise produced by ignitions, commutators, and microwave appliances;
  - thermal noise due to internal spontaneous fluctuations of electrons at the front end of individual receivers.

White spaces (for sure) and grey spaces (to a lesser extent) are obvious candidates for use by unserved operators. Of course, black spaces are to be avoided whenever and wherever the RF emitters residing in them are switched ON. However, when at a particular geographic location those emitters are switched OFF and the black spaces assume the new role of "spectrum holes," cognitive radio provides the opportunity for creating significant "white spaces" by invoking its dynamic-coordination capability for spectrum sharing, on which more is said in Section X.

### A. Detection Statistics

From these notes, it is apparent that a *reliable strategy for the detection of spectrum holes* is of paramount importance to the design and practical implementation of cognitive radio systems. Moreover, in light of the material presented in Section IV, the multitaper method combined with singular-value decomposition, hereafter referred to as the *MTM-SVD method*,<sup>13</sup> provides the method of choice for solving this detection problem by virtue of its accuracy and near-optimality.

By repeated application of the MTM-SVD method to the RF stimuli at a particular geographic location and from one burst of operation to the next, a time-frequency distribution of that location is computed. The dimension of time is quantized into discrete intervals separated by the burst duration. The dimension of frequency is also quantized into discrete intervals separated by resolution bandwidth of the multitaper spectral estimation procedure.

Let  $L$  denote the number of largest eigenvalues considered to play important roles in estimating the interference temperature, with  $|\sigma_l(f, t)|^2$  denoting the  $l$ th largest eigenvalue produced by the burst of RF stimuli received at time  $t$ . Let  $M$  denote the number of frequency resolutions of width  $\Delta f$ , which occupy the black space or gray space under scrutiny. Then, setting the discrete frequency

$$f = f_{\text{low}} + v \cdot \Delta f, \quad v = 0, 1, \dots, M-1$$

where  $f_{\text{low}}$  denotes the lowest end of a black/grey space, we may define the *decision statistic* for detecting the transition from such a space into a white space (i.e., spectrum hole) as

$$D(t) = \sum_{l=0}^{L-1} \sum_{v=0}^{M-1} |\sigma_l(f_{\text{low}} + v \cdot \Delta f; t)|^2 \Delta f. \quad (6)$$

<sup>13</sup>Mann and Park [40] discuss the application of the MTM-SVD method to the detection of oscillatory spatial-temporal signals in climate studies. They show that this new methodology avoids the weaknesses of traditional signal-detection techniques. In particular, the methodology permits a faithful reconstruction of spatio-temporal patterns of narrowband signals in the presence of additive spatially correlated noise.

Spectrum-hole detection is declared if two conditions are satisfied.

- 1) The reduction in  $D(t)$  from one burst to the next exceeds a prescribed threshold on several successive bursts.
- 2) Once the transition is completed,  $D(t)$  assumes minor fluctuations typical of ambient noise.

For a more refined approach, we may use an adaptive filter for *change detection* [42], [43]. Except for a scaling factor, the decision statistic  $D(t)$  provides an estimate of the interference temperature as it evolves with time  $t$  discretized in accordance with the burst duration. The adaptive filter is designed to produce a *model* for the time evolution of  $D(t)$  when the RF emitter responsible for the black space is switched ON. Assuming that the filter is provided with a sufficient number of adjustable parameters and the adaptive process makes it possible for the filter to produce a good fit to the evolution of  $D(t)$  with time  $t$ , the sequence of residuals produced by the model would ideally be the sample function of a *white noise* process. Of course, this state of affairs would hold only when the emitter in question is switched ON. Once the emitter is switched OFF, thereby setting the stage for the creation of a spectrum hole, the whiteness property of the model output disappears, which, in turn, provides the basis for detecting the transition from a black space into a spectrum hole. Whichever approach is used, the change-detection procedure would clearly have to be *location-specific*. For example, if the detection is performed in the basement of a building, the change in  $D(t)$  from a black space to a white space is expected to be significantly smaller than in an open environment. In any event, the detection procedure would have to be *sensitive* enough to work satisfactorily, regardless of location.

### B. Practical Issues Affecting the Detection of Spectrum Holes

The effort involved in the detection of spectrum holes and their subsequent exploitation in the management of radio spectrum should not be underestimated. In practical terms, the task of spectrum management (discussed in Section X) must not only be impervious to the modulation formats of primary users, but also several other issues.<sup>14</sup>

- 1) *Environmental factors*: Radio propagation across a wireless channel is known to be affected by the following factors.
  - *Path loss*, which refers to the diminution of received signal power with distance between the transmitter and the receiver.
  - *Shadowing*, which causes the received signal power to fluctuate about the path loss by a multiplication factor, thereby resulting in “coverage” holes.
- 2) *Exclusive zones*: An exclusion zone refers to the area (i.e., circle with some radius centered on the location of a primary user) inside which the spectrum is free of use and can, therefore, be made available to an unserved operator. This issue requires special attention in two possible scenarios.
  - The primary user happens to operate outside the exclusion zone, in which case the identification of a

spectrum hole must not be sensitive to radio interference produced by the primary user.

- Wireless scenarios built around *cooperative relay (ad hoc) networks* [45], [46], which are designed to operate at very low transmit powers. The dynamic spectrum management algorithm must be able to cope with such weak scenarios.
- 3) *Predictive capability for future use*: The identification of a spectrum hole at a particular geographic location and a particular time will only hold for that particular time and not necessarily for future time. Accordingly, the dynamic spectrum management algorithm in the transmitter must include two provisions.
    - Continuous monitoring of the spectrum hole in question.
    - Alternative spectral route for dealing with the eventuality of the primary user needing the spectrum for its own use.

## VI. CHANNEL-STATE ESTIMATION AND PREDICTIVE MODELING

As with every communication link, computation of the channel capacity of a cognitive radio link requires knowledge of *channel-state information* (CSI). This computation, in turn, requires the use of a procedure for estimating the state of the channel.

To deal with the channel-state estimation problem, traditionally, we have proceeded in one of two ways [47].

- *Differential detection*, which lends itself to implementation in a straightforward fashion to the use of  $M$ -ary phase modulation.
- *Pilot transmission*, which involves the periodic transmission of a pilot (training sequence) known to the receiver.

The use of differential detection offers robustness and simplicity of implementation, but at the expense of a significant degradation in the frame-error rate (FER) versus signal-to-noise ratio (SNR) performance of the receiver. On the other hand, pilot transmission offers improved receiver performance, but the use of a pilot is wasteful in both transmit power and channel bandwidth, the very thing we should strive to avoid. What then do we do, if the receiver requires knowledge of CSI for efficient receiver performance? The answer to this fundamental question lies in the use of *semi-blind training* of the receiver [48], which distinguishes itself from the differential detection and pilot transmission procedures in that the receiver has two modes of operations.

- 1) *Supervised training mode*: During this mode, the receiver acquires an estimate of the channel estimate, which is performed under the supervision of a short training sequence (consisting of two to four symbols) known to the receiver; the short training sequence is sent over the channel for a limited duration by the transmitter prior to the actual data transmission session.
- 2) *Tracking mode*: Once a reliable estimate of the channel state has been achieved, the training sequence is switched off, actual data transmission is initiated, and the receiver is switched to the tracking mode; this mode of operation

<sup>14</sup>The issues summarized herein follow a white paper submitted by Motorola to the FCC [44].

is performed in an unsupervised manner on a continuous basis during the course of data transmission.

### A. Channel Tracking

The evolution of CSI with time is governed by a *state-space model* comprised of two equations [48].

#### 1) Process equation:

The state of a wireless link is defined as the *minimal set of data on the past behavior of the link that is needed to predict the future behavior of the link*. For the sake of generality, we consider a *multiple-input–multiple-output (MIMO) wireless link*<sup>15</sup> of a narrowband category. Let  $x_{jk,t}$  denote the channel coefficient from the  $k$ th transmit antenna to the  $j$ th receive antenna at time  $t$ , with  $k = 1, 2, \dots, N_t$  and  $j = 1, 2, \dots, N_r$ . We may then describe the scalar form of the state equation as

$$x_{jk,t+1} = \sum_{l=0}^p \beta_{l,t} x_{jk,t-l} + d_{jk,t} \quad (7)$$

where the  $\beta_{l,t}$  are *time-varying autoregressive (AR) coefficients* and  $d_{jk,t}$  is the corresponding *dynamic noise*, both at time  $t$ . The AR coefficients account for the *memory* of the channel due to the multipath phenomenon. The upper limit of summation in (7) namely,  $p$ , is the *model order*. (The symbol  $p$  used here should not be confused with the symbol  $p$  used to denote the time-bandwidth product in Section III.)

#### 2) Measurement equation:

The measurement equation for the MIMO wireless link, also in scalar form, is described by

$$y_{j,t} = \sum_{k=1}^{N_t} s_{k,t} x_{jk,t} + v_{j,t} \text{ for } j = 1, 2, \dots, N_r \quad (8)$$

where  $s_{k,t}$  is the *encoded symbol* transmitted by the  $k$ th antenna at time  $t$ , and  $v_{j,t}$  is the corresponding *measurement noise* at the input of  $j$ th receive antenna at time  $t$ . The  $y_{j,t}$  is the *signal observed* at the output of the  $j$ th antenna at time  $t$ .

<sup>15</sup>The use of a MIMO link offers several important advantages [47].

- *Spatial degree of freedom*, defined by  $N = \min\{N_t, N_r\}$ , where  $N_t$  and  $N_r$  denote the numbers of transmit and receive antennas, respectively [49].
- *Increased spectral efficiency*, which is asymptotically defined by [49]

$$\lim_{N \rightarrow \infty} \frac{C(N)}{N} = \text{constant}$$

where  $C(N)$  is the ergodic capacity of the link, expressed as a function of  $N_t = N_r = N$ . This asymptotic property provides the basis for a spectacular increase in spectral efficiency by increasing the number of transmit and receive antennas.

- *Diversity*, which is asymptotically defined by [50]

$$\lim_{\rho \rightarrow \infty} \frac{\log \text{FER}(\rho)}{\log \rho} = -d_o$$

where  $d_o$  is the diversity order, and  $\text{FER}(\rho)$  is the frame-error rate expressed as a function of the SNR  $\rho$ .

These benefits (gained at the expense of increased complexity) commend the use of MIMO links for cognitive radio, all the more so considering the fact that the primary motivation for cognitive radio is the attainment of improved spectral efficiency. Simply put, a MIMO wireless link is not a necessary ingredient for cognitive radio but a highly desirable one.

The state-space model comprised of (7) and (8) is *linear*. The property of linearity is justified in light of the fact that the propagation of electromagnetic waves across a wireless link is governed by Maxwell's equations that are inherently linear.

What can we say about the AR coefficients, the dynamic noise, and measurement noise, which collectively characterize the state-space model of (7) and (8)? The answers to these questions determine the choice of an appropriate tracking strategy. In particular, the discussion of this issue addressed in [48] is summarized here.

- 1) *AR model*: A *Markovian model* of order  $p$  offers sufficient accuracy to model a Rayleigh-distributed time-varying channel.
- 2) *Noise processes*: The dynamic noise in the process equation and the measurement noise in the measurement equation can both assume *non-Gaussian* forms.

The finding reported under point 1) directly affects the design of the *predictive model*, which is an essential component of the channel tracker. The findings reported under point 2) prompt the search for a tracker outside of the classical Kalman filters, whose theory is rooted in Gaussian statistics.

A tracker that can operate in a non-Gaussian environment is the *particle filter*, whose theory is rooted in Bayesian estimation and Monte Carlo simulation [51], [52]. Each particle in the filter may be viewed as a Kalman filter merely in the sense that its operation encompasses two updates:

- state update;
- measurement update;

which bootstrap on each other, thereby forming a closed feedback loop. The particles are associated with *weights*, evolving from one iteration to the next. In particular, whenever the few particles whose weights assume negligible values, they are dropped from the computation. Thereafter, the filter concentrates on particles with large weights. In particular, on the next iteration of the filter, each of those particles is split into new particles whose multiplicity is determined in accordance with the weights of the parent particles. From this brief description, it is apparent that the computational complexity of a particle filter is in excess of that of a Kalman filter, but the particle filter makes up for it by being readily amenable to parallel computation.

In [48], the superior performance of the particle filter over the classical Kalman filter and other trackers (in the context of wireless channels) is demonstrated for real-life data. In light of the detailed studies reported in [48], we may conclude that the semi-blind estimation procedure, embodying the combined use of supervised training and channel tracking, offers an effective and efficient method for the extraction of channel-state estimation for use in a cognitive radio system.

The predictive AR model used in [48] is considered to be time-invariant (i.e., static) in that the model parameters are determined off-line (i.e., prior to transmission) and remain fixed throughout the tracking process. However, recognizing that a wireless channel is in actual fact nonstationary, with the degree of nonstationarity being highly dependent on the environ-



ment, we intuitively would expect that an improvement in performance of the channel tracker is achievable if somehow the predictive model is made *time-varying* (i.e., *dynamic*). This expectation has been demonstrated experimentally in [53] using MIMO wireless data. Specifically, the dynamic channel tracker accommodates a time-varying wireless channel by modeling the channel parameters themselves as *random walks*, thereby allowing them to assume a time-varying form.

Naturally, the maintenance of tracking a wireless channel in a reliable manner is affected by conditions of the channel. To be specific, we have found experimentally that when in the case of a MIMO wireless communication system the determinant of the channel matrix goes near zero, the particle filter experiences difficulty in tracking the channel. The reason for this phenomenon is that when the channel cannot support the information rate being used, the receiver makes too many symbol errors consecutively. This undesirable situation, in turn, causes the particle filter and, therefore, the receiver to lose track. Monitoring of the determinant of the channel matrix may, therefore, provide the means to prevent the loss of channel tracking.

### B. Rate Feedback

Channel-state estimation is needed by the receiver for coherent detection of the transmitted signal. Channel-state estimation is also needed for calculation of the channel capacity required by the transmitter for transmit-power control, which is to be discussed in Section IX. To satisfy this latter requirement, the receiver first uses Shannon's *information capacity theorem* to calculate the *instantaneous channel capacity*  $C$ , but rather than send  $C$  directly, the practical approach is to *quantize*  $C$  and feed the quantized transmission rate back to the transmitter, hence, the term *rate feedback*. A selection of quantized transmission rates is kept in a predetermined list, in which case the receiver picks the closest entry in the list that is less than the calculated value of  $C$  [54]; it is that particular entry in the list that forms the rate feedback.

In wireless communications, we typically find that there are significant fluctuations in the transmission rate. A *transmission-rate fluctuation* is considered to be significant if it is a predetermined fixed percentage of the mean rate for the channel. In any event, the transmitter would like to know the transmission-rate fluctuations. In particular, if the transmission rate is greater than the channel capacity, then there would be an *outage*. Correspondingly, the *outage capacity* is defined as the maximum bit rate that can be maintained across the wireless link for a prescribed probability of outage.

There are two other points to keep in mind.

- 1) *Rate-feedback delay*: There is always some finite time-delay in transmitting the quantized rate across the feedback channel. During the rate-feedback delay, the channel capacity  $C$  would inevitably vary, raising the potential possibility for an outage by picking too high a transmission rate. To mitigate this problem, prediction of the outage capacity becomes a necessary requirement, hence, the need for building a *predictive model* into the design of rate-feedback system in the receiver [55].
- 2) *Higher order Markov model*: Typically, a first-order Markov model is used to calculate the outage capacity of a MIMO wireless system. By definition, a first-order

Markov model relies on information gained from the state immediately preceding the current state; in other words, information pertaining to other previous states is considered to be of negligible importance. This assumption, usually made for mathematical tractability, is justified for a slow-fading wireless link. However, in the more difficult case of a fast-fading wireless link, the channel fluctuates more rapidly, which means that a higher order (e.g., second-order) Markov model is likely to provide more useful information about the current state than a first-order Markov model. Moreover, as the diversity order is increased, the channel becomes *hardened* quickly, in that variance of the channel capacity, relative to its mean, decreases rapidly [54]. For this same reason, we expect the fractional information gain about the current state due to the use of a higher order model to increase with decreasing diversity order [55].

## VII. COOPERATION AND COMPETITION IN MULTIUSER COGNITIVE RADIO ENVIRONMENTS

In this section, we set the stage for the next important task: transmit-power control.

In conventional wireless communications built around base stations, transmit-power levels are controlled by the base stations so as to provide the required coverage area and thereby provide the desired receiver performance. On the other hand, it may be necessary for a cognitive radio to operate in a *decentralized* manner, thereby broadening the scope of its applications. In such a case, some alternative means must be found to exercise control over the transmit power. The key question is: how can transmit-power control be achieved at the transmitter?

A partial answer to this fundamental question lies in building *cooperative mechanisms* into the way in which *multiple access* by users to the cognitive radio channel is accomplished. The cooperative mechanisms may include the following.

- 1) *Etiquette and protocol*. Such provisions may be likened to the use of traffic lights, stop signs, and speed limits, which are intended for motorists (using a highly dense transportation system of roads and highways) for their individual safety and benefits.
- 2) *Cooperative ad hoc networks*. In such networks, the users communicate with each other without any fixed infrastructure. In [45], Shepard studies a large packet radio network using spread-spectrum modulation. The only required form of coordination in the network is that of *pairwise between neighboring nodes (users) that are in direct communication*. To mitigate interference, it is proposed that each node create a transmit-receive schedule. The schedule is communicated to a nearest neighbor only when a source node's schedule and that of the neighboring node permit the source node to transmit it and the neighboring node to receive it. Under some reasonable assumptions, simulations are presented to show that with this completely *decentralized control*, the network can scale to almost arbitrary numbers of nodes.

In an independent and like-minded study [46], Gupta and Kumar considered a radio network consisting of  $n$  identical nodes that communicate with each other. The

nodes are arbitrarily located inside a disk of unit area. A data packet produced by a source node is transmitted to a sink node (i.e., destination) via a series of hops across intermediate nodes in the network. Let one *bit-meter* denote one bit of information transmitted across a distance of one meter toward its destination. Then, the *transport capacity* of the network is defined as the total number of bit-meters that the network can transport in one second for all  $n$  nodes. Under a protocol model of noninterference, Gupta and Kumar derive two significant results. First, the transport capacity of the network increases with  $n$ . Second, for a node communicating with another node at a distance nonvanishingly far away, the throughput (in bits per second) decreases with increasing  $n$ . These results are consistent with those of Shephard. However, Gupta and Kumar do not consider the congestion problem identified in Shephard's work.

Through the cooperative mechanisms described under 1) and 2) and other cooperative means, the users of cognitive radio may be able to benefit from cooperation with each other in that the system could end up being able to support more users because of the potential for an improved spectrum-management strategy.

The cooperative ad hoc networks studied by Shepard [45] and Gupta and Kumar [46] are examples of a new generation of wireless networks, which, in a loose sense, resemble the Internet. In any event, in cognitive radio environments built around ad hoc networks and existing infrastructured networks, it is possible to find the multiuser communication process being complicated by another phenomenon, namely, *competition*, which works in opposition to cooperation.

Basically, the driving force behind competition in a multiuser environment lies in having to operate under the umbrella of *limitations imposed on available network resources*. Given such an environment, a particular user may try to exploit the cognitive radio channel for self-enrichment in one form or another, which, in turn, may prompt other users to do likewise. However, exploitation via competition should not be confused with the *self-orientation* of cognitive radio which involves the assignment of priority to certain stimuli (e.g., urgent requirements or needs). In any event, the control of transmit power in a multiuser cognitive radio environment would have to operate under two stringent limitations on network resources: the interference-temperature limit imposed by regulatory agencies, and the availability of a limited number of spectrum holes depending on usage. What we are describing here is a multiuser communication-theoretic problem. Unfortunately, a complete understanding of *multiuser communication theory* is yet to be developed. Nevertheless, we know enough about two diverse disciplines, namely, information theory and game theory, for us to tackle this difficult problem in a meaningful way. However, before proceeding further, we digress briefly to introduce some basic concepts in game theory.

## VIII. STOCHASTIC GAMES

The transmit-power control problems in a cognitive-radio environment (involving multiple users) may be viewed as a

*game-theoretic problem*.<sup>16</sup> In the absence of competition, we would then have an entirely cooperative game, in which case the problem simplifies to an optimal control-theoretic problem. This simplification is achieved by finding a single cost function that is optimized by all the players, thereby eliminating the game-theoretic aspects of the problem [58]. So, the issue of interest is how to deal with a *noncooperative game* involving multiple players. To formulate a mathematical framework for such an environment, we have to account for three basic realities:

- a state space that is the product of the individual players' states;
- state transitions that are functions of *joint actions* taken by the players;
- payoffs to individual players that depend on joint actions as well.

That framework is found in *stochastic games* [57], which, also occasionally appear under the name "Markov games" in the computer science literature.

A stochastic game is described by the five-tuple  $\{\mathcal{N}, \mathcal{S}, \vec{\mathcal{A}}, \mathcal{P}, \vec{\mathcal{R}}\}$ , where

- $\mathcal{N}$  is a set of players, indexed  $1, 2, \dots, n$ ;
- $\mathcal{S}$  is a set of possible states;
- $\vec{\mathcal{A}}$  is the *joint-action space* defined by the product set  $\mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_n$ , where  $\mathcal{A}_j$  is the set of actions available to the  $j$ th player;
- $\mathcal{P}$  is a probabilistic transition function, an element of which for joint action  $a$  satisfies the condition

$$\sum_{s' \in \mathcal{S}} p_{ss'}^a = 1 \quad \text{for all } s \in \mathcal{S} \text{ and } a \in \vec{\mathcal{A}}$$

- $\vec{\mathcal{R}} = r_1 \times r_2 \times \dots \times r_n$ , where  $r_j$  is the payoff for the  $j$ th player and which is a function of the joint actions of all  $n$  players.

One other notational issue: the action of player  $j \in \mathcal{N}$  is denoted by  $a_j$ , while the joint actions of the other  $n - 1$  players in the set  $\mathcal{N}$  are denoted by  $a_{-j}$ . We use a similar notation for some other variables.

Stochastic games are *supersets* of two kinds of decision processes, namely, Markov decision process and matrix games, as illustrated in Fig. 2. A *Markov decision process* is a special case of a stochastic game with a single player, that is,  $n = 1$ . On the other hand, a *matrix game* is a special case of a stochastic game with a single state, that is,  $|\mathcal{S}| = 1$ .

### A. Nash Equilibria and Mixed Strategies

With two or more *players*<sup>17</sup> being an integral part of a *game*, it is natural for the study of cognitive radio to be motivated by certain ideas in game theory. Prominent among those ideas for *finite games* (i.e., stochastic games for which each player has only a finite number of alternative courses of action) is that of a Nash equilibrium, so named for the Nobel Laureate John Nash.

<sup>16</sup>In a historical context, the formulation of game theory may be traced back to the pioneering work of John von Neumann in the 1930s, which culminated in the publication of the coauthored book entitled "Theory of Games and Economic Behavior" [56]. For modern treatments of game theory, see the books under [57] and [58].

<sup>17</sup>Players are referred to as *agents* in the machine learning literature.

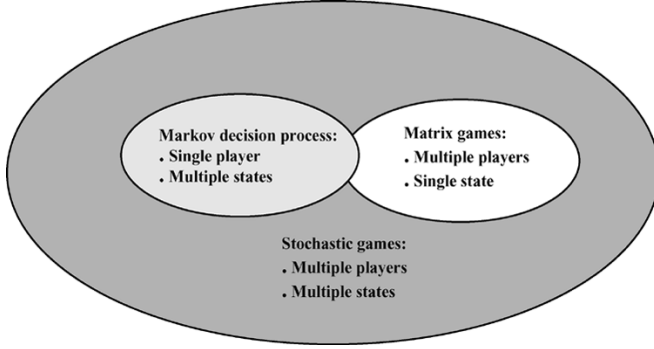


Fig. 2. Highlighting the differences between Markov decision processes, matrix games, and stochastic games.

A *Nash equilibrium* is defined as an action profile (i.e., vector of players' actions) in which each action is a best response to the actions of all the other players [59]. According to this definition, a Nash equilibrium is a *stable operating* (i.e., *equilibrium*) point in the sense that there is no incentive for any player involved in a finite game to change strategy given that all the other players continue to follow the equilibrium policy. The important point to note here is that the Nash-equilibrium approach provides a powerful tool for modeling nonstationary processes. Simply put, it has had an enormous influence on the evolution of game theory by shifting its emphasis toward the study of equilibria as a predictive concept.

With the learning process modeled as a *repeated stochastic game* (i.e., repeated version of a one-shot game), each player gets to know the past behavior of the other players, which may influence the current decision to be made. In such a game, the task of a player is to select the best mixed strategy, given information on the mixed strategies of all other players in the game; hereafter, other players are referred to as "opponents." A *mixed strategy* is defined as a *continuous randomization by a player of its own actions*, in which the actions (i.e., pure strategies) are selected in a deterministic manner. Stated in another way, *the mixed strategy of a player is a random variable whose values are the pure strategies of that player*.

To explain what we mean by a mixed strategy, let  $a_{j,k}$  denote the  $k$ th action of player  $j$  with  $k = 1, 2, \dots, K$ . The mixed strategy of player  $j$ , denoted by the set of probabilities  $\{p_{j,k}\}_{k=1}^K$ , is an integral part of the linear combination

$$q_j = \sum_{k=1}^K p_{j,k} a_{j,k} \quad \text{for } j = 1, 2, \dots, n. \quad (9)$$

Equivalently, we may express  $q_j$  as the inner product

$$q_j = \mathbf{p}_j^T \mathbf{a}_j \quad \text{for } j = 1, 2, \dots, n \quad (10)$$

where

$$\mathbf{p}_j = [p_{j,1}, p_{j,2}, \dots, p_{j,K}]^T$$

is the *mixed strategy vector*, and

$$\mathbf{a}_j = [a_{j,1}, a_{j,2}, \dots, a_{j,K}]^T$$

is the *deterministic action vector*. The superscript  $T$  denotes matrix transposition. For all  $j$ , the elements of the mixed strategy vector  $\mathbf{p}_j$  satisfy the following two conditions:

1)

$$0 \leq p_{j,k} \leq 1. \quad (11)$$

2)

$$\sum_{k=1}^K p_{j,k} = 1. \quad (12)$$

Note also that the mixed strategies for the different players are *statistically independent*.

The motivation for permitting the use of mixed strategies is the well-known fact that every stochastic game has *at least one Nash equilibrium* in the space of mixed strategies but not necessarily in the space of pure strategies, hence, the preferred use of mixed strategies over pure strategies. The purpose of a learning algorithm is that of computing a mixed strategy, namely a sequence  $\{q^{(1)}, q^{(2)}, \dots, q^{(t)}\}$  over time  $t$ .

It is also noteworthy that the implication of (9) through (12) is that the entire set of mixed strategies lies inside a *convex simplex* or *convex hull*, whose dimension is  $K - 1$  and whose  $K$  vertices are the  $a_{j,k}$ . Such a geometric configuration makes the selection of the best mixed strategy in a multiple-player environment a more difficult proposition to tackle than the selection of the best base action in a single-player environment.

### B. Limitations of Nash Equilibrium

The formulation of Nash equilibrium assumes that the players are *rational*, which means that each player has a "view of the world." According to Aumann and Brandenburger [60], mutual knowledge of rationality and common knowledge of beliefs is sufficient for deductive justification of the Nash equilibrium. *Belief* refers to state of the world, expressed as a set of probability distributions over tests; by "tests" we mean a sequence of actions and observations that are executed at a specific time.

Despite the insightful value of the Aumann–Brandenburger exposition, the notion of the Nash equilibrium has two practical limitations.

- 1) The approach advocates the use of a *best-response strategy* (i.e., a strategy whose outcome against an opponent with a similar goal is the best possible one), but in a two-player game for example, if one player adopts a nonequilibrium strategy, then the optimal response of the other player is of a nonequilibrium kind too. In such situations, the Nash-equilibrium approach is no longer applicable.
- 2) Description of a noncooperative game is essentially confined to an equilibrium condition; unfortunately, the approach does not teach us about the underlying dynamics involved in establishing that equilibrium.

To refine the Nash equilibrium theory, we may embed *learning models* in the formulation of game-theoretic algorithms. This new approach provides a foundation for equilibrium theory, in which less than fully rational players strive for some form of optimality over time [57], [61].

### C. Game-Theoretic Learning: No-Regret Algorithms

*Statistical learning theory* is a well-developed discipline for dealing with *uncertainty*, which makes it well-suited for solving game-theoretic problems. In this context, a class of *no-regret*

*algorithms* is attracting a great deal of attention in the machine-learning literature.

The provision of “no-regret” is motivated by the desire to ensure two practical end-results.

- 1) A player does not get unlucky in an arbitrary nonstationary environment. Even if the environment is not adversarial, the player could experience bad performance when using an algorithm that assumes independent and identically distributed (i.i.d.) examples; the no-regret provision guarantees that such a situation does not arise.
- 2) Clever opponents of that player do not exploit dynamic changes or limited resources for their own selfish benefits.

The notion of regret can be defined in different ways.<sup>18</sup> One particular definition of no regret is basically a rephrasing of *boosting*, the original formulation of which is due to Freund and Schapire [62]. Basically, boosting refers to the training of a committee machine in which several experts are trained on data sets with entirely different distributions [62], [71]. It is a general method that can be used to improve the performance of *any* learning model. Stated in another way, boosting provides a method for modifying the underlying distribution of examples in such a way that a strong learning model is built around a set of weak learning modules.

To see how boosting can also be viewed as a no-regret proposition, consider a prediction problem with  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{t-1}$  denoting the sequence of input vectors. Let  $\hat{\mathbf{x}}_t$  denote the one-step prediction at time  $t$  computed by the boosting algorithm operating on this sequence. The prediction error is defined by the difference  $\mathbf{e}_t = \mathbf{x}_t - \hat{\mathbf{x}}_t$ . Let  $l(\mathbf{e}_t)$  denote a convex cost function of the prediction error  $\mathbf{e}_t$ ; the mean-square error is an example of such a cost function. After processing  $N$  examples, the resulting cost function of the boosting algorithm is given by

$$L_N = \sum_{t=1}^N l(\mathbf{e}_t). \quad (13)$$

If, however, the prediction was to be performed by one of the experts using some fixed hypothesis  $h$  to yield the prediction error  $\mathbf{e}_t(h)$ , the corresponding cost function would have the value

$$L_N(h) = \sum_{t=1}^N l(\mathbf{e}_t(h)). \quad (14)$$

The regret for not having used hypothesis  $h$  is the difference

$$\rho_N(h) = L_N - L_N(h) = \sum_{t=1}^N l(\mathbf{e}_t) - l(\mathbf{e}_t(h)). \quad (15)$$

We say that the regret is *negative* if the difference  $\rho_N(h)$  is negative. Let  $H$  denote the class of all hypotheses used in the algorithm. Then the overall regret for not having used the best hypothesis  $h \in H$  is given by the supremum

$$\rho_N = \sup_{h \in H} \rho_N(h). \quad (16)$$

<sup>18</sup>In a unified treatment of game-theoretic learning algorithms, Greenwald [61] identifies three regret variations:

- External regret
- Internal regret
- Swap regret

External regret coincides with the notion of boosting as defined by Freund and Schapire [62].

A boosting algorithm is synonymous with no-regret algorithms because the overall regret  $\rho_N$  is small no matter which particular sequence of input vectors is presented to the algorithm.

Unfortunately, most no-regret algorithms are designed on the premise that the hypotheses are chosen from a small, discrete set, which, in turn, limits applicability of the algorithms. To overcome this limitation, Gordon [63] expands on the Freund-Schapire boosting (Hedge) algorithm by considering a class of prediction problems with *internal structure*. Specifically, the internal structure presumes two things.

- 1) The input vectors are assumed to lie on or inside an almost arbitrary *convex* set, so long as it is possible to perform *convex optimization*; for example, we could have a  $d$ -dimensional polyhedron or  $d$ -dimensional sphere, where  $d$  is dimensionality of the input space.
- 2) The prediction rules (i.e., experts) are purposely designed to be linear.

An example scenario that has the internal structure embodied under points 1) and 2) is that of planning in a stochastic game described by a Markov decision process, in which state-action costs are controlled by an adversarial or clever opponent after the player in question fixes its own policy. The reader is referred to [64] for such an example involving a robot path-planning problem, which may be likened to a cognitive radio problem made difficult by the actions of a clever opponent.

Given such a framework, we can always make a legal prediction in an efficient manner via *convex duality*, which is an inherent property of convex optimization [65]. In particular, it is always possible to choose a legal hypothesis that prevents the total regret from growing too quickly (and, therefore, causes the average regret to approach zero).

By exploiting this internal structure, Gordon derives a new learning rule referred to as the *Lagrangian hedging algorithm* [63]. This new algorithm is of a gradient descent kind, which includes two steps, namely, projection and scaling. The *projection step* simply ensures that we always make a legal prediction. The *scaling step* adaptively adjusts the degree to which the algorithm operates in an aggressive or conservative manner. In particular, if the algorithm predicts poorly, then the cost function assumes a large value on the average, which, in turn, tends to make the predictions change slowly.

The algorithm derives its name from a combination of two points.

- 1) The algorithm depends on one free parameter, namely, a *convex hedging function*.
- 2) The hypothesis of interest can be viewed as a *Lagrange multiplier* that keeps the regret from growing too fast.

To expand on the Lagrangian issue under point 2), consider the case of a matrix game using a regret-matching algorithm. *Regret-matching*, embodied in the *generalized Blackwell condition* [61], means that the probability distribution over actions by a player is proportional to the positive elements in the regret vector of that player. For example, in the so-called “rock-scissors-paper” game in which rock smashes scissors, scissors cut paper, and paper wraps the rock, if we currently have a vector made up as follows:

- regret 2 versus rock;
- regret  $-7$  versus scissors;
- regret 1 versus paper;

then we would play rock 2/3 of the time, never play scissors, and play paper 1/3 of the time. The prediction at each step of the regret-matching algorithm is a probability distribution over actions. Ideally, we desire the *no-regret property*, which means that the *average* regret vector approaches the region where all of its elements are less than or equal to zero. However, at any finite time, in practice, the regret vector may still have positive elements. (The magnitudes of these elements are bounded by theorems presented in [63].) In such a situation, we cannot achieve the no-regret condition exactly in finite time. Rather, we apply a *soft constraint* by imposing a *quadratic penalty function* on each positive element of the regret vector. The penalty function involves the sum of two components, one being the *hedging function* and the other being an *indicator function* for the set of unnormalized hypotheses using a gradient vector. The *gradient vector* is itself defined as the derivative of the penalty function with respect to the regret vector, the evaluation being made at the current regret vector. It turns out that the gradient vector is just the regret vector with all negative elements set equal to zero. The desired hypothesis is gotten by normalizing this vector to form a probability distribution of actions, which yields exactly the regret-matching algorithm. In choosing the distribution of actions in the manner described herein, we enforce the constraint that the regret vector is not allowed to move upwards along the gradient. *Gordon's gradient descent theorem*, proved by induction in [63], shows that the quadratic penalty function cannot grow too quickly, which in turn, means that our average gradient vector will get closer to the negative orthant, as desired.

In short, the Lagrangian hedging algorithm is a no-regret algorithm designed to handle internal structure in the set of allowable predictions. By exploiting this internal structure, tight bounds on performance and fast rates of convergence are achieved when the provision of no regret is of utmost importance.

## IX. DISTRIBUTED TRANSMIT-POWER CONTROL: ITERATIVE WATER-FILLING

As an alternative to game-theoretic learning exemplified by a no-regret algorithm, we may look to another approach, namely, water-filling (WF) rooted in information theory [66]. To be specific, consider a cognitive radio environment involving  $n$  transmitters and  $n$  receivers. The environmental model is based on two assumptions.

- 1) Communication across a channel is asynchronous, in which case the communication process can be viewed as a *noncooperative game*. For example, in a *mesh network* consisting of a mixture of ad hoc networks and existing infrastructured networks, the communication process from a base station to users is controlled in a synchronous manner, but the multihop communication process across the ad hoc network could be asynchronous and, therefore, noncooperative.
- 2) A *signal-to-noise ratio (SNR) gap* is included in calculating the transmission rate so as to account for the gap between the performance of a practical coding-modulation scheme and the theoretical value of channel capacity.

(In effect, the SNR gap is large enough to assure reliable communication under operating conditions all the time.)

In mathematical terms, the essence of transmit-power control for such a noncooperative multiuser radio environment is stated as follows.

*Given a limited number of spectrum holes, select the transmit-power levels of  $n$  unserved users so as to jointly maximize their data-transmission rates, subject to the constraint that the interference-temperature limit is not violated.*

It may be tempting to suggest that the solution of this problem lies in simply increasing the transmit-power level of each unserved transmitter. However, increasing the transmit-power level of any one transmitter has the undesirable effect of also increasing the level of interference to which the receivers of all the other transmitters are subjected. The conclusion to be drawn from this reality is that it is not possible to represent the overall system performance with a single index of performance. (This conclusion further confirms what we said previously in Section VIII.) Rather, we have to adopt a *tradeoff* among the data rates of all unserved users in some computationally tractable fashion.

Ideally, we would like to find a *global solution* to the constrained maximization of the joint set of data-transmission rates under study. Unfortunately, finding this global solution requires an exhaustive search through the space of all possible power allocations, in which case we find that the computational complexity needed for attaining the global solution assumes a prohibitively high level.

To overcome this computational difficulty, we use a new optimization criterion called *competitive optimality*<sup>19</sup> for solving the transmit-power control problem, which may now be restated as follows.

*Considering a multiuser cognitive radio environment viewed as a noncooperative game, maximize the performance of each unserved transceiver, regardless of what all the other transceivers do, but subject to the constraint that the interference-temperature limit not be violated.*

This formulation of the distributed transmit-power control problem leads to a solution that is of a *local* nature; though sub-optimum, the solution is insightful, as described next.

### A. Two-User Scenario: Simultaneous WF is Equivalent to Nash Equilibrium

Consider the simple scenario of Fig. 3 involving two users communicating across a flat-fading channel. The complex-valued baseband channel matrix is denoted by

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}. \quad (17)$$

Viewing this scenario as a noncooperative game, we may describe the two players of the game as follows:

<sup>19</sup>The competitive optimality criterion is discussed in Yu's doctoral dissertation [67, Ch. 4]. In particular, Yu develops an iterative WF algorithm for a sub-optimum solution to the multiuser digital subscriber line (DSL) environment, viewed as a noncooperative game.

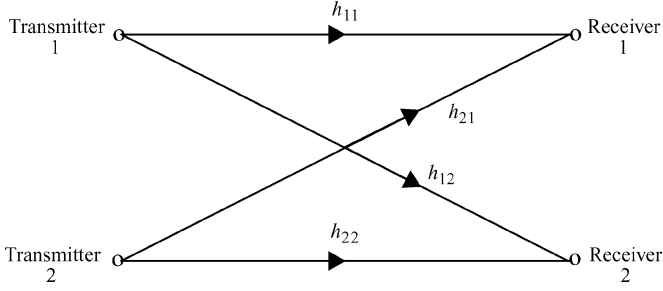


Fig. 3. Signal-flow graph of a two-user communication scenario.

- The two *players*<sup>20</sup> are represented by *transmitters* 1 and 2.
- The pure strategies (i.e., deterministic actions) of the two players are defined by the *power spectral densities*  $S_1(f)$  and  $S_2(f)$  that, respectively, pertain to the transmitted signals radiated by the antennas of transmitters 1 and 2.
- The *payoffs* to the two players are defined by the *data-transmission rates*  $R_1$  and  $R_2$ , which are, respectively, produced by transmitters 1 and 2.

From the discussions presented in Section IV, we recognize that the noise floor of the RF radio environment is characterized by a frequency-dependent parameter: the power spectral density  $S_N(f)$ . In effect,  $S_N(f)$  defines the “noise floor” above which the transmit-power controller must fit the transmission-data requirements of users 1 and 2.

Define the *cross-coupling* between the two users in terms of two new real-valued parameters  $\alpha_1$  and  $\alpha_2$  by writing

$$\alpha_1 = \frac{\Gamma|h_{12}|^2}{|h_{22}|^2} \quad (18)$$

and

$$\alpha_2 = \frac{\Gamma|h_{21}|^2}{|h_{11}|^2} \quad (19)$$

where  $\Gamma$  is the SNR gap. Assuming that the receivers do *not* perform any form of interference-cancellation irrespective of the received signal strengths, we may, respectively, formulate the achievable data-transmission rates  $R_1$  and  $R_2$  as the two definite integrals

$$R_1 = \int_{\text{hole } 1} \log_2 \left( 1 + \frac{S_1(f)}{N_1(f) + \alpha_2 S_2(f)} \right) df \quad (20)$$

and

$$R_2 = \int_{\text{hole } 2} \log_2 \left( 1 + \frac{S_2(f)}{N_2(f) + \alpha_1 S_1(f)} \right) df. \quad (21)$$

The term  $\alpha_2 S_2(f)$  in the first denominator and the term  $\alpha_1 S_1(f)$  in the second denominator are due to the cross-coupling between the transmitters and receivers. The remaining two terms  $N_1(f)$  and  $N_2(f)$  are noise terms defined by

$$N_1(f) = \frac{\Gamma S_{N,1}(f)}{|h_{11}|^2} \quad (22)$$

<sup>20</sup>In the two-user example of Fig. 3, each user is represented by a single-input–single-output (SISO) wireless system—hence, the adoption of transmitters 1 and 2 of the two systems as the two players in a game-theoretic interpretation of the example. In a MIMO generalization of this example, each user has multiple transmitters. Nevertheless, there are still two players, with the two players being represented by the two sets of multiple transmitters.

and

$$N_2(f) = \frac{\Gamma S_{N,2}(f)}{|h_{22}|^2} \quad (23)$$

where  $S_{N,1}(f)$  and  $S_{N,2}(f)$  are, respectively, the particular parts of the noise-floor’s spectral density  $S_N(f)$  that define the spectral contents of spectrum holes 1 and 2. We are now ready to formally state the competitive optimization problem as follows.

*Given that the power spectral density  $S_2(f)$  of transmitter 2 is fixed, maximize the transmission-data  $R_1$  of (20), subject to the constraint*

$$\int_{\text{hole } 1} [S_1(f) + N_1(f) + \alpha_2 S_2(f)] df \leq kT_{\max}$$

where  $T_{\max}$  is the prescribed interference-temperature limit and  $k$  is Boltzmann’s constant. A similar statement applies to the competitive optimization of transmitter 2.

Of course, it is understood that both  $S_1(f)$  and  $S_2(f)$  remain nonnegative for all  $f$ . The solution to the optimization problem described herein follows the allocation of transmit power in accordance with the WF procedure [66], [67].

Fig. 4 presents the results of an experiment<sup>21</sup> on the two-user wireless scenario, which were obtained using the WF procedure. To add meaning to the important result portrayed in Fig. 4, we may state that the optimal competitive response to the all pure-strategy corresponds to a Nash equilibrium. Stated in another way, *a Nash equilibrium is reached if, and only if, both users simultaneously satisfy the WF condition* [67].

An assumption implicit in the WF solution presented in Fig. 4 is that each transmitter of cognitive radio has knowledge of its position with respect to the receivers in its operating range at all times. In other words, cognitive radio has *geographic awareness*, which is implemented by embedding a *global positioning*

<sup>21</sup>Specifications of the experiment presented in Fig. 4 are as follows. Narrowband channels (uniformly spaced in frequency) available to the two users:

- user 1: channels 1, 2, and 3;
- user 2: channels 4, 5, and 6.

Modulation Strategy: orthogonal frequency-division multiplexing (OFDM)  
Multiuser path-loss matrix

0.5207	0	0	0.0035	0.0020	0.0024
0	0.5223	0	0.0030	0.0034	0.0031
0	0	0.5364	0.0040	0.0015	0.0035
0.0036	0.0002	0.0023	0.7136	0	0
0.0028	0.0029	0.0011	0	0.6945	0
0.0022	0.0010	0.0034	0	0	0.7312

Target data transmission rates:

- user 1: 9 bits/symbol;
- user 2: 12 bits/symbol;

Power constraint (imposed by interference-temperature limit) = 0 dB.

Receiver noise-power level = −30 dB.

Ambient interference power level = −24 dB.

The solution presented in Fig. 4 is achieved in two iterations of the WF algorithm.

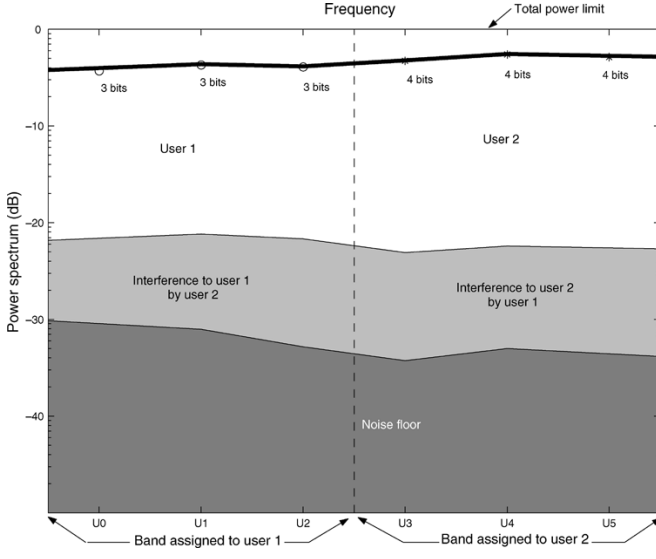


Fig. 4. Two-user profile, illustrating two things. 1) The spectrum-sharing process performed using the iterative WF algorithm. 2) The bit-loading curve shown “bold-faced” at the top of the figure.

satellite (GPS) receiver in the system design [68]. The transmitter puts its geographic awareness to good use by calculating the path loss incurred in the course of electromagnetic propagation of the transmitted signal to each receiver in the transmitter’s operating range, which, in turn, makes it possible to calculate the *multiuser path-loss matrix* of the environment.<sup>22</sup>

### B. Multiuser Scenario: Iterative WF Algorithm

Emboldened by the WF solution illustrated in Fig. 4 for a two-user scenario, we may formulate an *iterative two-loop WF algorithm* for the distributed transmit-power control of a multiuser radio environment. The environment involves a set of transmitters indexed by  $i = 1, 2, \dots, n$  and a corresponding set of receivers indexed by  $j = 1, 2, \dots, n$ . Viewing the multiuser radio environment as a non cooperative game and assuming the availability of an adequate number of spectrum holes to accommodate the *target* data-transmission rates, the algorithm proceeds as follows [67].

- 1) *Initialization*: Unless some prior knowledge is available, the power distribution across the  $n$  users is set equal to zero.

<sup>22</sup>Let  $d$  denote the distance from a transmitter to a receiver. Extensive measurements of the electromagnetic field strength, expressed as a function of the distance  $d$ , carried out in various radio environments have motivated an *empirical propagation formula for the path loss*, which expresses the received signal power  $P_R$  in terms of the transmitted signal power  $P_T$  as follows [47]:

$$P_R = \left( \frac{\beta}{d^m} \right) P_T$$

where the *path-loss exponent*  $m$  varies from 2 to 5, depending on the environment, and the *attenuation parameter*  $\beta$  is frequency-dependent.

Considering the general case of  $n$  transmitters indexed by  $i$ , and  $n$  receivers indexed by  $j$ , let  $h_{ij}$  denote the complex-valued channel coefficient from transmitter  $i$  to receiver  $j$ . Then, in light of the empirical propagation formula, we may write

$$|h_{ij}|^2 = \frac{P_{R,j}}{P_{T,i}} = \frac{\beta}{d_{ij}^m}, \quad i = 1, 2, \dots, n \quad j = 1, 2, \dots, n$$

where  $d_{ij}$  is the distance from transmitter  $i$  to receiver  $j$ . Hence, knowing  $\beta$ ,  $m$ , and  $d_{ij}$  for all  $i$  and  $j$ , we may calculate the multiuser path-loss matrix.

- 2) *Inner loop (iteration)*: Given a set of allowed channels (i.e., spectrum-holes):
  - User 1 performs WF, subject to its power constraint. At first, the user employs one channel; but if its target rate is not satisfied, it will try to employ two channels, and so on. The WF by user 1 is performed with only the noise floor to account for.
  - Then, user 2 performs the WF process, subject to its own power constraint. At this point, in addition to the noise floor, the WF computation accounts for interference produced by user 1.
  - The power-constrained WF process is continued until all  $n$  users are dealt with.
- 3) *Outer loop (iteration)*: After the inner iteration is completed, the power allocation among the  $n$  users is adjusted:
  - If the actual data-transmission rate of any user is found to be greater than its target value, the transmit power of that user is reduced.
  - If, on the other hand, the actual data-transmission rate of any user is less than the target value, the transmit power is increased, keeping in mind that the interference temperature limit is not violated.
- 4) *Confirmation step*: After the power adjustments, up or down, are completed, the transmission-data rates of all the  $n$  users are checked:
  - If the target rates of all the  $n$  users are satisfied, the computation is terminated.
  - Otherwise, the algorithm goes back to the inner loop, and the computations are repeated. This time, however, the WF performed by every user, including user 1, must account for the interference produced by all the other users.

In effect, the outer loop of the distributed transmit-power controller tries to find the minimum level of transmit power needed to satisfy the target data-transmission rates of all  $n$  users.

For the distributed transmit-power controller to function properly, two requirements must be satisfied.

- Each user knows, *a priori*, its own target rate.
- All the target rates lie within a *permissible rate region*; otherwise, some or all of the users will violate the interference-temperature limit.

To distributively live within the permissible rate region, the transmitter needs to be equipped with a *centralized agent* that has knowledge of the channel capacity (through rate-feedback from the receiver) and multiuser path-loss matrix (by virtue of geographic awareness). The centralized agent is thereby enabled to decide which particular sets of target rates are indeed attainable.

### C. Iterative WF Algorithm Versus No-Regret Algorithm

The iterative WF approach, rooted in communication theory, has a “top-down, dictatorially controlled” flavor. In contrast, a no-regret algorithm, rooted in machine learning, has a “bottom-up” flavor. In more specific terms, we may make the following observations.

- 1) The iterative WF algorithm exhibits fast-convergence behavior by virtue of incorporating information on both the

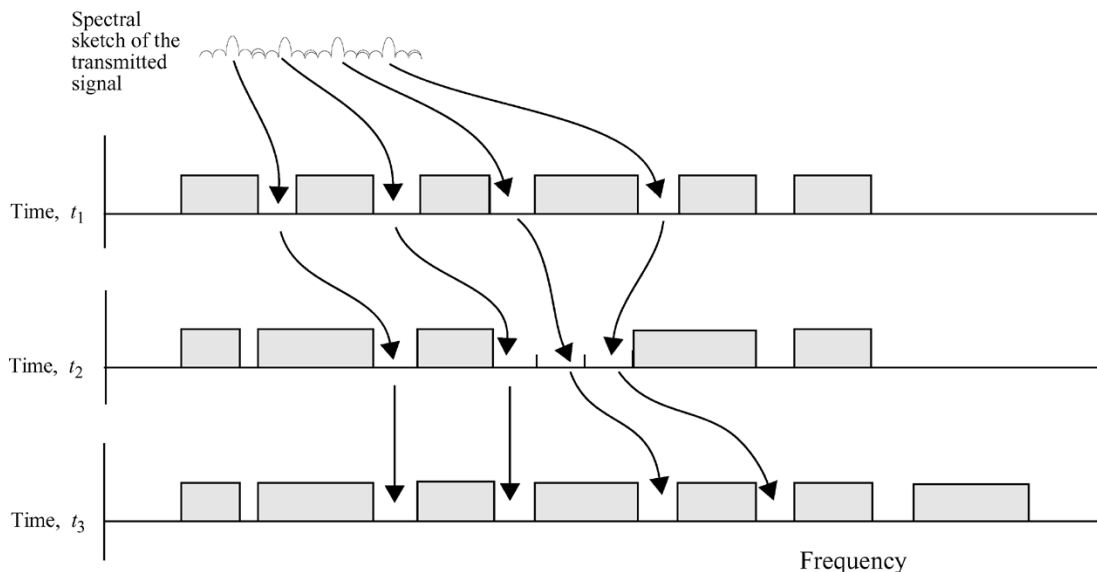


Fig. 5. Illustrating the notion of dynamic spectrum-sharing for OFDM based on four channels, and the way in which the spectrum manager allocates the requisite channel bandwidths for three time instants  $t_1 < t_2 < t_3$ , depending on the availability of spectrum holes.

channel and RF environment. On the other hand, a no-regret algorithm exemplified by the Lagrangian hedging algorithm relies on first-order gradient information, causing it to converge comparatively slowly.

- 2) The Lagrangian hedging learner has the attractive feature of incorporating a regret agenda, the purpose of which is to guarantee that the learner cannot be deceptively exploited by a clever player. On the other hand, the iterative WF algorithm lacks a learning strategy that could enable it to guard against exploitation.

In short, the iterative WF approach has much to offer for dealing with multiuser scenarios, but its performance could be improved through interfacing with a more competitive, regret-conscious learning machine that enables it to mitigate the exploitation phenomenon.

## X. DYNAMIC SPECTRUM MANAGEMENT

As with transmit-power control, dynamic spectrum management (also referred to as dynamic frequency-allocation) is performed in the transmitter. Indeed, these two tasks are so intimately related to each other that we have included them both inside a single functional module, which performs the role of *multiple-access control* in the basic cognitive cycle of Fig. 1.

Simply put, the primary purpose of spectrum management is to develop an adaptive strategy for the efficient and effective utilization of the RF spectrum. Specifically, the *spectrum-management algorithm* is designed to do the following.

*Building on the spectrum holes detected by the radio-scene analyzer and the output of transmit-power controller, select a modulation strategy that adapts to the time-varying conditions of the radio environment, all the time assuring reliable communication across the channel.*

Communication reliability is assured by choosing the SNR gap  $\Gamma$  large enough as a design parameter, as discussed in Section IX.

### A. Modulation Considerations

A modulation strategy that commends itself to cognitive radio is the OFDM<sup>23</sup> by virtue of its flexibility and computational efficiency. For its operation, OFDM uses a set of carrier frequencies centered on a corresponding set of narrow channel bandwidths. Most important, the availability of rate feedback (through the use of a feedback channel) permits the use of *bit-loading*, whereby the number of bits/symbol for each channel is optimized for the SNR characterizing that channel; this operation is illustrated by the bold-faced curve in Fig. 4.

As time evolves and spectrum holes come and go, the bandwidth-carrier frequency implementation of OFDM is dynamically modified, as illustrated in the time-frequency picture in Fig. 5 for the case of four carrier frequencies. The picture illustrated in Fig. 5 describes a distinctive feature of cognitive radio: a *dynamic spectrum-sharing process*, which evolves in time. In effect, the spectrum-sharing process satisfies the constraint imposed on cognitive radio by the availability of spectrum holes at a particular geographic location and their possible variability with time. Throughout the spectrum-sharing process, the transmit-power controller keeps an account of the bit-loading across the spectrum holes in use. In effect, the dynamic spectrum manager and the transmit-power controller work in concert together, thereby fulfilling the multiple-access control requirement.

Starting with a set of spectrum holes, it is possible for the dynamic spectrum management algorithm to confront a situation where the prescribed FER cannot be satisfied. In situations of this kind, the algorithm can do one of two things:

- 1) work with a more spectrally efficient modulation strategy, or else;
- 2) incorporate the use of another spectrum hole, assuming availability.

<sup>23</sup>OFDM has been standardized; see the IEEE 802.16 Standard, described in [69].



In approach 1), the algorithm resorts to increased computational complexity, and in approach 2), it resorts to increased channel bandwidth so as to maintain communication reliability.

### B. Traffic Considerations

In a code-division multiple-access (CDMA) system, like the IS-95, there is a phenomenon called *cell breathing*: the cells in the system effectively shrink and grow over time [70]. Specifically, if a cell has more users, then the interference level tends to increase, which is counteracted by allocating a new incoming user to another cell; that is, the cell coverage is shrunk. If, on the other hand, a cell has less users, then the interference level is correspondingly lowered, in which case the cell coverage is allowed to grow by accommodating new users. So in a CDMA system, the traffic and interference levels are associated together. In a cognitive radio system, based on CDMA, the dynamic spectrum management algorithm naturally focuses on the allocation of users, first to white spaces with low interference levels, and then to grey spaces with higher interference levels.

When using other multiple-access techniques, such as OFDM, co-channel interference must be avoided. To satisfy this requirement, the dynamic-spectrum management algorithm must include a *traffic model* of the primary user occupying a black space. The traffic model, built on historical data, provides the means for predicting the future traffic patterns in that space. This in turn, makes it possible to predict the duration for which the spectrum hole vacated by the incumbent primary user is likely to be available for use by a cognitive radio operator.

In a wireless environment, two classes of traffic data patterns are distinguished, as summarized here.

- 1) *Deterministic patterns*. In this class of traffic data, the primary user (e.g., TV transmitter, radar transmitter) is assigned a fixed time slot for transmission. When it is switched OFF, the frequency band is vacated and can, therefore, be used by a cognitive radio operator.
- 2) *Stochastic patterns*. In this second class, the traffic data can only be described in statistical terms. Typically, the *arrival times* of data packets are modeled as a *Poisson process* [70]; while the service times are modeled as *exponentially distributed*, depending on whether the data are of packet-switched or circuit-switched kind, respectively. In any event, the model parameters of stochastic traffic data vary slowly and, therefore, lend themselves to on-line estimation using historical data. Moreover, by building a *tracking strategy* into design of the predictive model, the accuracy of the model can be further improved.

## XI. EMERGENT BEHAVIOR OF COGNITIVE RADIO

The cognitive radio environment is naturally time varying. Most important, it exhibits a unique combination of characteristics (among others): adaptivity, awareness, cooperation, competition, and exploitation. Given these characteristics, we may wonder about the emergent behavior of a cognitive radio environment in light of what we know on two relevant fields: *self-organizing systems*, and *evolutionary games*.

First, we note that the emergent behavior of a cognitive radio environment viewed as a game, is influenced by the *degree of*

*coupling* that may exist between the actions of different players (i.e., transmitters) operating in the game. The coupling may have the effect of *amplifying* local perturbations in a manner analogous with *Hebb's postulate of learning*, which accounts for self-amplification in self-organizing systems [71]. Clearly, if they are left unchecked, the amplifications of local perturbations would ultimately lead to *instability*. From the study of self-organizing systems, we know that competition among the constituents of such a system can act as a stabilizing force [71]. By the same token, we expect that competition among the users of cognitive radio for limited resources (e.g., spectrum holes) may have the influence of a *stabilizer*.

For additional insight, we next look to evolutionary games. The idea of evolutionary games, developed for the study of ecological biology, was first introduced by Maynard Smith in 1974. In his landmark work [72], [73], Smith wondered whether the theory of games could serve as a tool for modeling conflicts in a population of animals. In specific terms, two critical insights into the emergence of so-called *evolutionary stable strategies* were presented by Smith, as succinctly summarized in [74] and [75].

- The animals' behavior is stochastic and unpredictable, when it is viewed at the microscopic level of individual acts.
- The theory of games provides a plausible basis for explaining the complex and unpredictable patterns of the animals' behavior.

Two key issues are raised here.

- 1) *Complexity*:<sup>24</sup> The emergent behavior of an evolutionary game may be *complex*, in the sense that a change in one or more of the parameters in the underlying dynamics of the game can produce a dramatic change in behavior. Note that the dynamics must be nonlinear for complex behavior to be possible.
- 2) *Unpredictability*. Game theory does not require that animals be fundamentally unpredictable. Rather, it merely requires that the individual behavior of each animal be *unpredictable with respect to its opponents* [73], [74].

From this brief discussion on evolutionary games, we may conjecture that the emergent behavior of a multiuser cognitive radio environment is explained by the unpredictable action of each user, as seen individually by the other users (i.e., opponents).

Moreover, given the conflicting influences of cooperation, competition, and exploitation on the emergent behavior of a cognitive radio environment, we may identify two possible end-results [81].

- 1) *Positive emergent behavior*, which is characterized by *order* and, therefore, a harmonious and efficient utilization of the radio spectrum by all users of the cognitive

<sup>24</sup>The new sciences of complexity (whose birth was assisted by the Santa Fe Institute, New Mexico) may well occupy much of the intellectual activities in the 21st century [76]–[78]. In the context of complexity, it is perhaps less ambiguous to speak of complex behavior rather than complex systems [79]. A nonlinear dynamical system may be complex in computational terms but incapable of exhibiting complex behavior. By the same token, a nonlinear system can be simple in computational terms but its underlying dynamics are rich enough to produce complex behavior.

radio. (The positive emergent behavior may be likened to Maynard Smith's evolutionary stable strategy.)

- 2) *Negative emergent behavior*, which is characterized by *disorder* and, therefore, a culmination of traffic jams, chaos,<sup>25</sup> and unused radio spectrum.

From a practical perspective, what we need are, first, a reliable criterion for the early detection of negative emergent behavior (i.e., disorder) and, second, corrective measures for dealing with this undesirable behavior. With regards to the first issue, we recognize that cognition, in a sense, is an exercise in assigning probabilities to possible behavioral responses, in light of which we may say the following. In the case of positive emergent behavior, predictions are possible with nearly complete confidence. On the other hand, in the case of negative emergent behavior, predictions are made with far less confidence. We may, thus, think of a *likelihood function* based on predictability as a criterion for the onset of negative emergent behavior. In particular, we envision a *maximum-likelihood detector*, the design of which is based on the predictability of negative emergent behavior.

## XII. DISCUSSION

Cognitive radio holds the promise of a new *frontier in wireless communications*. Specifically, with dynamic coordination of the spectrum-sharing process, significant "white space" can be created, which, in turn, makes it possible to improve spectrum utilization under *constantly changing user conditions* [82]. The dynamic spectrum-sharing capability builds on two matters.

- 1) Paradigm shift in wireless communications from transmitter-centricity to *receiver-centricity*, whereby interference power rather than transmitter emission is regulated.
- 2) *Awareness* of and *adaptation* to the environment by the radio.

### A. Language Understanding

Cognitive radio is a computer-intensive system, so much so that we may think of it as a "radio with a computer inside or a computer that transmits" [83]. The system provides a novel basis for balancing the communication and computing needs of a user against those of a network with which the user would like to operate. With so much reliance on computing, it is obvious that language understanding would play a key role in the organization of domain knowledge for the cognitive cycle, which includes the following [6].

- 1) *Wake cycle*, during which the cognitive radio supports the tasks of passive radio-scene analysis, channel-state estimation and predictive modeling, and active transmit-power control and dynamic spectrum management.
- 2) *Sleep cycle*, during which incoming stimuli are integrated into the domain knowledge of a "personal digital assistant."

<sup>25</sup>The possibility of characterizing negative emergent behavior as a chaotic phenomenon needs some explanation. Idealized chaos theory is based on the premise that dynamic noise in the state-space model (describing the phenomenon of interest) is zero [80]. However, it is unlikely that this highly restrictive condition is satisfied by real-life physical phenomena. So, the proper thing to say is that it is feasible for a negative emergent behavior to be *stochastic chaotic*.

- 3) *Prayer cycle*, which caters to items that cannot be dealt with during the sleep cycle and may, therefore, be resolved through interaction of the cognitive radio with the user in real time.

### B. Cognitive MIMO Radio

It is widely recognized that the use of a MIMO antenna architecture can provide for a spectacular increase in the spectral efficiency of wireless communications [47]. With improved spectrum utilization as one of the primary objectives of cognitive radio, it seems logical to explore building the MIMO antenna architecture into the design of cognitive radio. The end-result is a *cognitive MIMO radio* that offers the ultimate in flexibility, which is exemplified by four degrees of freedom: carrier frequency, channel bandwidth, transmit power, and multiplexing gain.

### C. Cognitive Turbo Processing

*Turbo processing* has established itself as one of the key technologies for modern digital communications [84]. In specific terms, turbo processing has made it possible to provide significant improvements in the signal-processing operations of channel decoding and channel equalization, both of which are basic to the design of digital communication systems. Compared with traditional design methodologies, these improvements manifest themselves in spectacular reductions in FERs for prescribed SNRs. With *quality-of-service* (QoS) being an essential requirement of cognitive radio, it also seems logical to build turbo processing into the design of cognitive radio.

### D. Nanoscale Processing

With computing being so central to the implementation of cognitive radio, it is natural that we keep *nanotechnology* [85] in mind as we look to the future. Since the observation of multiwalled carbon nanotubes for the first time in transmission electron microscopy studies in 1991 by Iijima [86], *carbon nanotubes* have been explored extensively in theoretical and experimental studies of nanotechnology [87], [88]. Most important, nanotubes offer the potential for a paradigm shift from the narrow confine of today's information processing based on silicon technology to a much broader field of information processing, given the rich *electromechano-optochemical functionalities* that are endowed in nanotubes [89]. This paradigm shift may well impact the evolution of cognitive radio in its own way.

### E. Concluding Remarks

The potential for cognitive radio to make a significant difference to wireless communications is immense, hence, the reference to it as a "disruptive, but unobtrusive technology." In the final analysis, however, the key issue that will shape the evolution of cognitive radio in the course of time, be that for civilian or military applications, is *trust*, which is two-fold [81], [90]:

- trust by the users of cognitive radio;
- trust by all other users who might be interfered with.

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